HIGH SCHOOL MATHEMATICS

INTEGRATION

The integrals you need to know:

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
- $\int kx^n dx = \frac{k}{n+1}\, x^{n+1} + C$

Algebraic Functions

- $\int e^x dx = e^x + C$
- $\int \frac{1}{x} dx = \ln|x| + C$

Exponentials & Logarithms

- $\int \cos(x)\, dx = \sin(x) + C$
- $\int \sin(x)\, dx = -\cos(x) + C$
- $\int \sec^2(x)\, dx = \tan(x) + C$
- $\int cosec(x)\cot(x)\, dx = -\operatorname{cosec}(x) + C$
- $\int \operatorname{cosec}^2(x)\, dx = -\cot(x) + C$
- $\int \sec(x)\tan(x)\, dx = \sec(x) + C$

Trigonometric

- $\int \sec^2(kx)\, dx = \frac{1}{k}\tan(kx) + C$
- $\int \tan(x)\, dx = \ln|\sec(x)| + C$
- $\int \sec(x)\, dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \cot(x)\, dx = \ln|\sin(x)| + C$
- $\int cosec(x) dx = -\ln|cosec(x) + \cot(x)| + C$

These trigonometric derivatives are provided in the booklet.

Volume of Revolution

$$Volume = \pi \int_a^b y^2\, dx$$

Volume of revolution formed by rotating y about the x-axis between x=a and x=b.

Integration by parts (provided in the booklet)

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

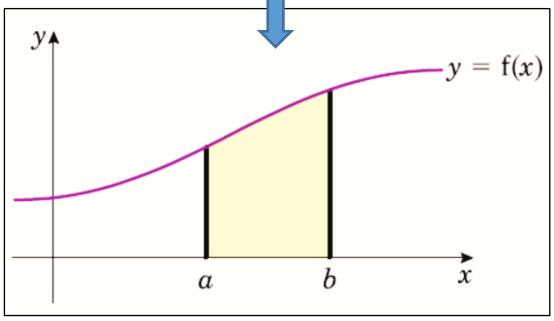
Trapezium rule (provided in the booklet)

$$\int_a^b y\, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1})\}, \textbf{where } h = \frac{b-a}{n}$$

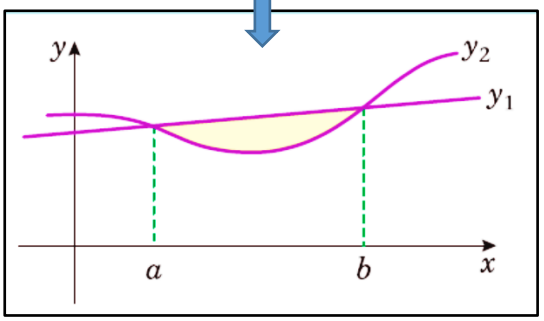
You may use chain rule in reverse to obtain the integrals given below.

- $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$
- $\int e^{(ax+b)} dx = \frac{1}{a} e^{(ax+b)} + C$
- $\int \frac{1}{(ax+b)} dx = \frac{1}{a} \ln|(ax+b)| + C$
- $\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b) + C$
- $\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b) + C$
- $\int \sec^2(ax+b)\, dx = \frac{1}{a}\tan(ax+b) + C$
- $\int cosec(ax+b)\cot(ax+b)\, dx = -\frac{1}{a}\,\text{cosec}\,(ax+b) + C$
- $\int \text{cosec}^2(ax+b)\, dx = -\frac{1}{a}\cot(ax+b) + C$
- $\int \sec(ax+b)\tan(ax+b)\, dx = \frac{1}{a}\sec(ax+b) + C$

$$A = \int_a^b y\, dx$$



$$A = \int_a^b (y_1 - y_2)\, dx$$



Some useful hints:

- ***Make use of trigonometric identities to help change the expression into one you know how to integrate. For example;***

$$\int \boldsymbol{sin^2(x)dx} = \int \left(\frac{\mathbf{1}}{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{cos}\,(\mathbf{2x})\right) \boldsymbol{dx}$$

- ***Use partial fractions to integrate expressions like;***

$$\frac{\boldsymbol{x+7}}{\boldsymbol{(x+2)(x-3)}}$$

- ***Simplify the expression by substitution method where you feel integrating directly is troublesome.***
- ***Remember the pattern in the following type of integrations;***

Natural logarithm

$$\int \frac{\boldsymbol{f'(x)}}{\boldsymbol{f(x)}}\, \boldsymbol{dx} = \ln|\boldsymbol{f(x)}| + \boldsymbol{C}$$

Powers of functions

$$\int \boldsymbol{f'(x)} \times [\boldsymbol{f(x)}]^{\boldsymbol{n}}\, \boldsymbol{dx} = \frac{\mathbf{1}}{\boldsymbol{n+1}}[\boldsymbol{f(x)}]^{\boldsymbol{n+1}}; \boldsymbol{n \neq -1}$$

- ***Parametric integrations; replace*** $[\,\boldsymbol{dx}\,]$ ***with*** $\left[\frac{\boldsymbol{dx}}{\boldsymbol{dt}}\,\boldsymbol{dt}\right]$ ***and follow the same procedure for integration by substitution .***