MHIGH SCHOOL MATHEMATICS

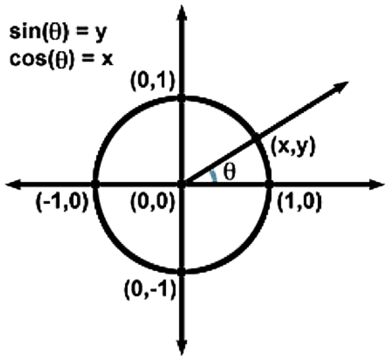
TRIGONOMETRY II

The equivalent ratios that you can the derive from the unit circle:

- $\sin(\theta) = \sin(180° - \theta)$
- $\sin(\theta) = \sin(\theta \pm 360°)$
- $\cos(\theta) = \cos(-\theta)$
- $\cos(\theta) = \cos(360° - \theta)$
- $\cos(\theta) = \cos(\theta \pm 360°)$
- $\tan(\theta) = \tan(\theta \pm 180°)$

Radian measures: θ in radians

- $\sin(\theta) = \sin(\pi - \theta)$
- $\sin(\theta) = \sin(\theta \pm 2\pi)$
- $\cos(\theta) = \cos(-\theta)$
- $\cos(\theta) = \cos(2\pi - \theta)$
- $\cos(\theta) = \cos(\theta \pm 2\pi)$
- $\tan(\theta) = \tan(\theta \pm \pi)$



Identities from AS

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

Boundaries

$$-1 \le \sin(\theta) \le 1$$

$$-1 \le \cos(\theta) \le 1$$

Identities related to sec & cosec

$$1 + tan^2(\theta) = sec^2(\theta)$$

$$1 + cot^2(\theta) = cosec^2(\theta)$$

Secant, cosecant and cotangent ratios

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad cosec(\theta) = \frac{1}{\sin(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

Addition Formulae (Provided in the booklet)

$$\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \times \tan B}$$

$$\tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \times \tan B}$$

Double angle formulae

$$\sin(2A) \equiv 2\sin A \cos A$$

$$\cos(2A) \equiv \cos^2 A - \sin^2 A$$

$$\cos(2A) \equiv 2\cos^2 A - 1$$

$$\cos(2A) \equiv 1 - 2\sin^2 A$$

$$\tan(2A) \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Factor formulae (Provided in the booklet)

$$\sin A + \sin B \equiv 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B \equiv 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B \equiv 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B \equiv -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

You can solve equations of the form $\boldsymbol{a\cos\theta + b\sin\theta = c}$ *, where a, b and c are constants and* $c \neq 0$*, by writing the left hand side as sine function only or a cosine function only***.**

For positive values of a and b,

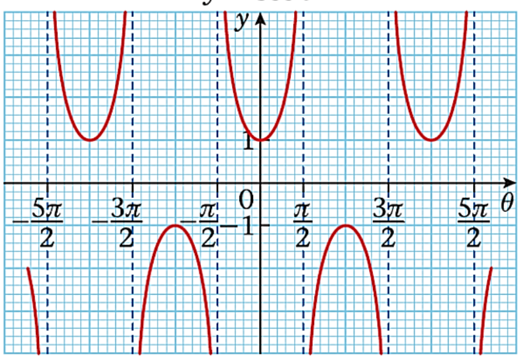
$\boldsymbol{a\sin\theta \pm b\cos\theta}$***, can be expressed in the form*** $\boldsymbol{R\sin(\theta \pm \alpha)}$ ***, with R>0 and*** $\boldsymbol{0 < \alpha < 90°}$***(or*** $\boldsymbol{\frac{\pi}{2}}$***)***

$\boldsymbol{a\cos\theta \pm b\sin\theta}$***, can be expressed in the form*** $\boldsymbol{R\cos(\theta \mp \alpha)}$ ***, with R>0 and*** $\boldsymbol{0 < \alpha < 90°}$***(or*** $\boldsymbol{\frac{\pi}{2}}$***)***

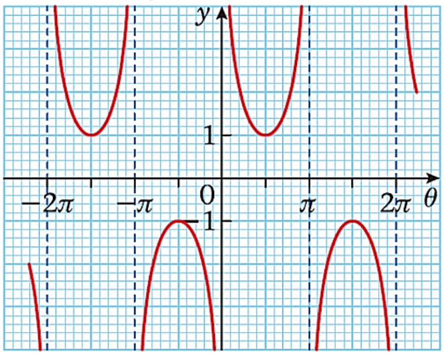
where $\boldsymbol{R\cos\alpha = a}$ ***and*** $\boldsymbol{R\sin\alpha = b}$ ***and*** $\boldsymbol{R = \sqrt{a^2 + b^2}}$ ***.***

Note : Rather than memorizing two different formulae while writing in this form ,focus on expanding $\boldsymbol{R\sin(\theta \pm \alpha)}$ **[or** $\boldsymbol{R\cos(\theta \mp \alpha)}$**],** and equalize it to given expression (simply match the formats).

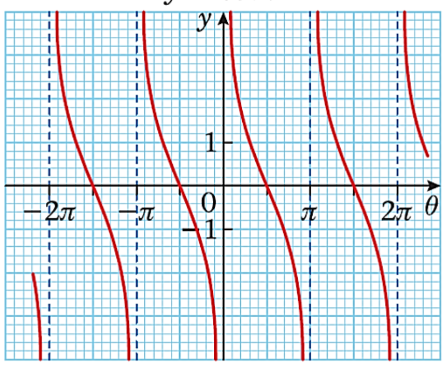
$y = \sec\theta$



$y = \operatorname{cosec}\theta$

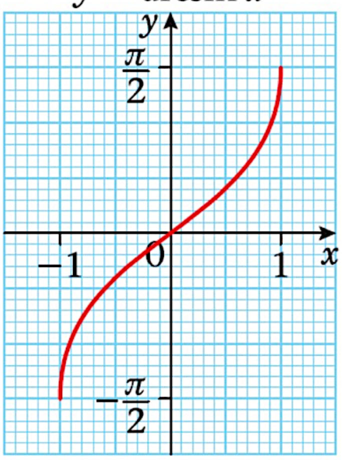


$y = \cot\theta$

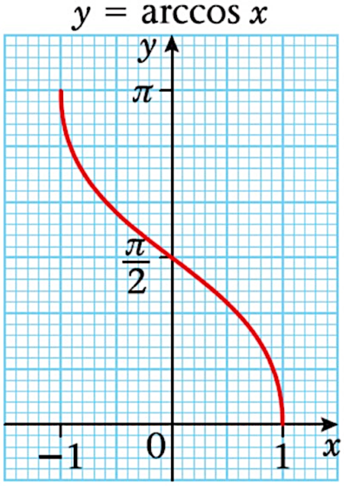


Inverse trigonometric functions:

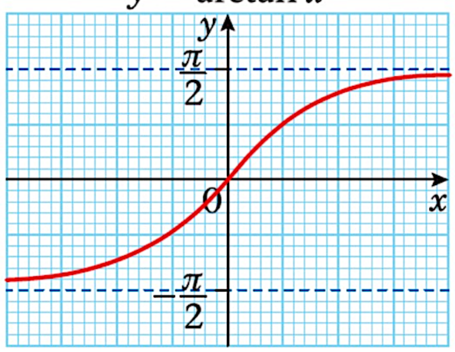
$y = \arcsin x$



$y = \arccos x$



$y = \arctan x$



- The inverse function of $\sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, is called $arc\sin x$; it has domain $-1 \leq x \leq 1$ and range $-\frac{\pi}{2} \leq arc\sin x \leq \frac{\pi}{2}$.
- The inverse function of $\cos x$, $0 \leq x \leq \pi$, is called $arc\cos x$; it has domain $-1 \leq x \leq 1$ and range $0 \leq arc\cos x \leq \pi$.
- The inverse function of $\tan x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, is called $arc\tan x$; it has domain $x \in \mathbb{R}$ and range $-\frac{\pi}{2} \leq arc\tan x \leq \frac{\pi}{2}$.